DIPOLE IN AN $\vec{E}$ - FIELD

Before solving this problem let us recall the relevant part of Phys 121.

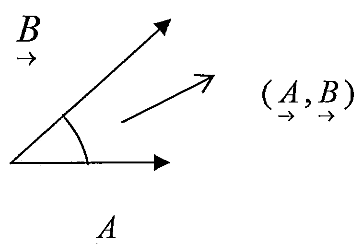
<u>1st</u>

CROSS-PRODUCT OF TWO VECTORS: Given two vectors $\vec{A}$ and $\vec{B}$, their cross-product or vector Product $\vec{C}$ is given by:

$$\vec{C} = \vec{A} X \vec{B}$$

Magnitude of $\vec{C}$

$$C = AB\,Sin(\vec{A},\vec{B})$$



Direction of $\vec{C}$ is normal to $(\vec{A},\vec{B})$ plane that is,

$$\vec{C} \perp \vec{A}$$

$$\vec{C} \perp \vec{B}$$

Supplemented by the right hand rule

<u>Take</u>

$\vec{A}$ || Thumb

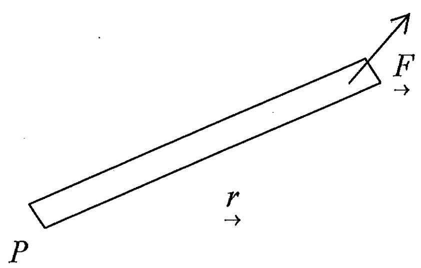
$\vec{B}$ || Fingers

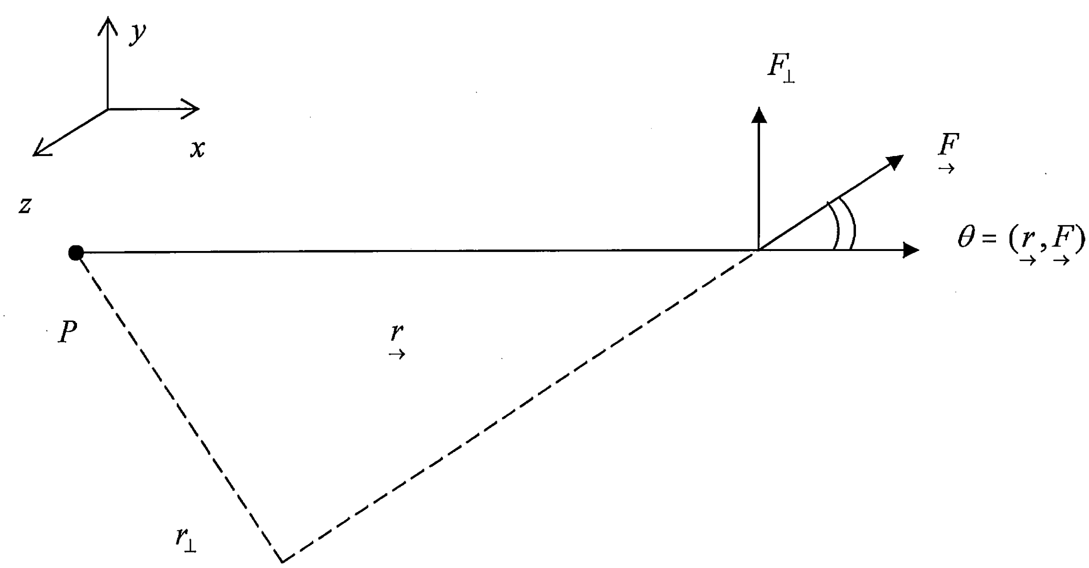
Then

$\vec{C}$ is $\perp$ palm of right hand

2^{nd}

TORQUE: When a force $\vec{F}$ is applied at some distance $\vec{r}$ from a pivot pt. P on an extended object such as a bar, the bar turns about the pivot point on an axis which is perpendicular both to $\vec{r}$ and $\vec{F}$, this turning is controlled by the torque (vector) $\vec{\tau}$. Just as a force is required to cause an object to change its linear momentum (center of mass velocity) Torque is needed to change angular velocity (angular momentum). FORCE CAUSES LINEAR ACCELERATION, (TRANSLATION) TORQUE CAUSES ANGULAR ACCELERATION (ROTATION).



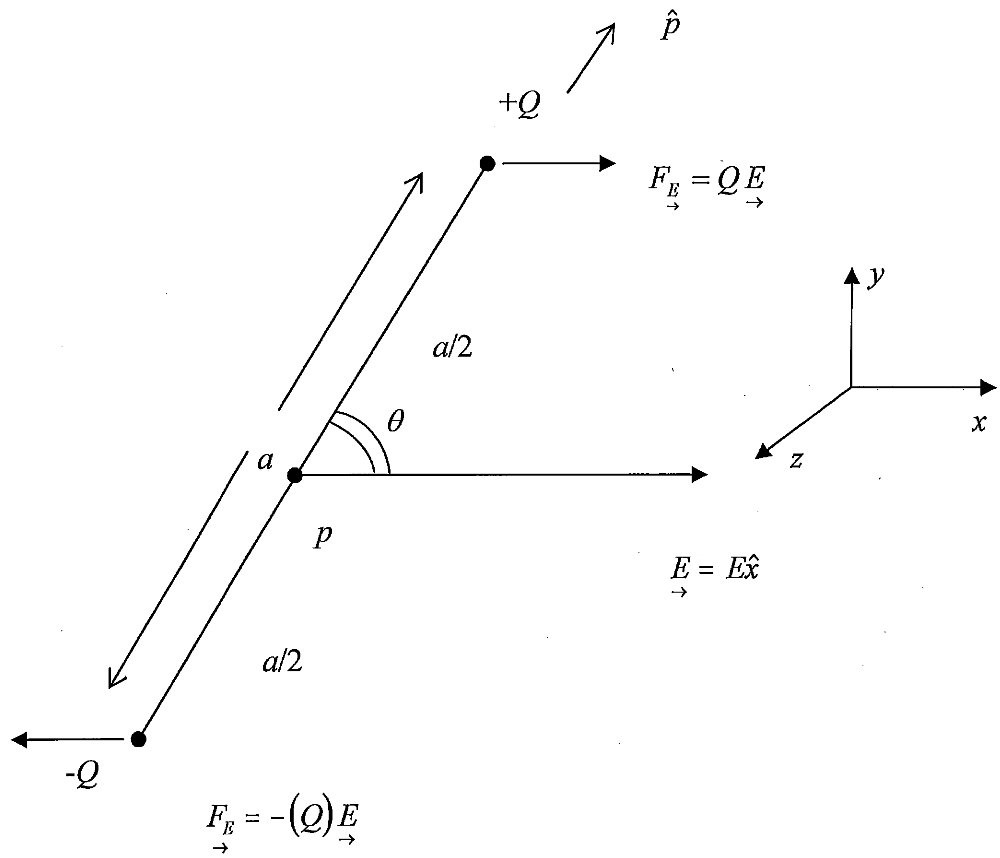


Notice that in turning the bar about an axis through P, only component of $\vec{F} \perp \vec{r}$ matters or alternatively, the moment arm $r_\perp$ controls τ.

$$\underset{\rightarrow}{\tau} = \underset{\rightarrow}{r} \, x \, \underset{\rightarrow}{F}$$
$$= rF \, Sin\theta\hat{z}$$

Thus magnitude of Torque is $\tau = rF_{\perp} = r_{\perp}F = rF\,Sin(\underset{\rightarrow}{r},\underset{\rightarrow}{F})$ with the right hand rule for the picture above

Now let us put a dipole $\underset{\rightarrow}{p} = aQ\hat{p}$ in an $\underset{\rightarrow}{E}$ field.



Notice there are two forces on the Dipole. $+Q\underset{\rightarrow}{E}$ & $-|Q|\underset{\rightarrow}{E}$, so total force is zero. Take torque about pivot point P (mid pt. Of Dipole). It is the sum of two torques.

Magnitude

$$\tau = \frac{a}{2}QE\,Sin\theta + \frac{a}{2}QE\,Sin\theta$$
$$= aQE\,Sin\theta$$
$$= pE\,Sin\theta$$

And for the geometry shown and using the right hand rule one can write $\underset{\rightarrow}{\tau} = \underset{\rightarrow}{p}\,X\,\underset{\rightarrow}{E}$

Because mag. is $pESin\theta$ & direction is $\perp$ paper pointing down! $\underset{\rightarrow}{\tau} = -pE\,Sin\theta\hat{z}$

Potential Energy of DIPOLE IN $\underset{\rightarrow}{E}-$ *field*.

Work done $\Delta W = \underset{\rightarrow}{\tau} \bullet \underset{\rightarrow}{\Delta\theta}$ $\quad [\Delta W = \underset{\rightarrow}{F} \bullet \underset{\rightarrow}{\Delta x}]$

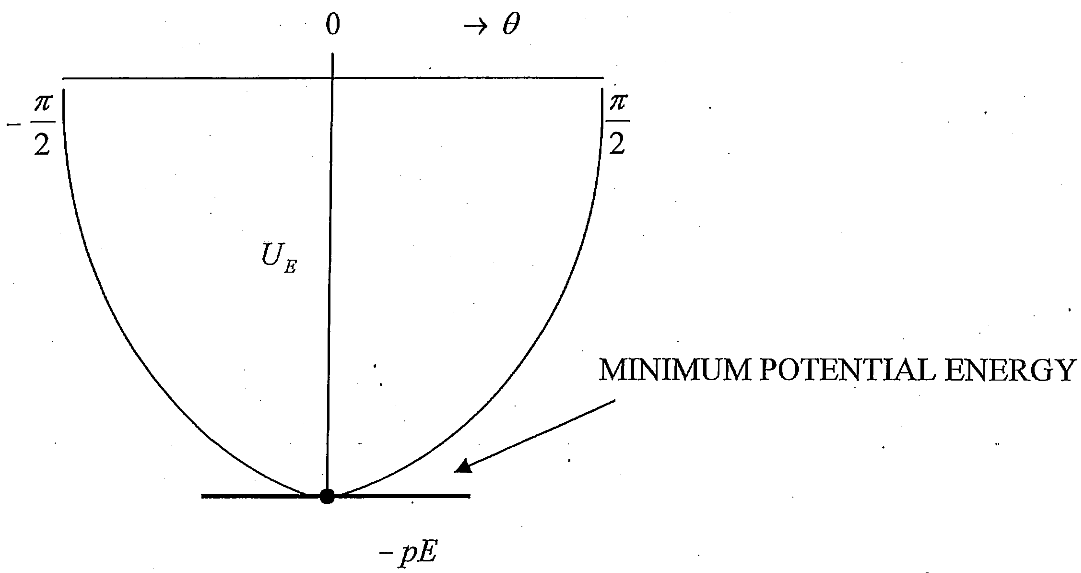
$\underset{\rightarrow}{\tau} = -pE\,Sin\theta\hat{z}$

$\underset{\rightarrow}{\Delta\theta} = \Delta\theta\hat{z}$

$\Delta W = -pE\,Sin\theta\Delta\theta$

Change of Potential Energy $\Delta U_E = -\Delta W = +pE\,Sin\,\theta\Delta\theta$
Hence $U_E = -(\underset{\rightarrow}{P} \bullet \underset{\rightarrow}{E}) = -pE\,Cos\theta$

SO DIPOLE WILL TURN AND BECOME PARALLEL To $\underset{\rightarrow}{E}$ field at $\equiv m$.



FINALLY, NOTE THAT FOR $\theta << 1$, $U_E \alpha \theta^2$ *or* $\underset{\rightarrow}{\tau}\,\alpha - \theta\hat{z}$, so you get Linear Harmonic Oscillations, that is, for small θ dipole will oscillate about $\underset{\rightarrow}{E}$. In other words, dipole is in $\equiv m$ when it is parallel to $\underset{\rightarrow}{E}$, if you pull it aside by a small angle θ and let go it will oscillate about $\underset{\rightarrow}{E}$.